Cambridge International Examinations
Cambridge International Advanced Level

CANDIDATE NAME

CENTRE NUMBER | CANDIDATE NUMBER

0610869629

FURTHER MATHEMATICS **9231/12**

Paper 1 **May/June 2018**

3 hours

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

Write your Centre number, candidate number and name in the spaces at the top of this page.
Write in dark blue or black pen.
You may use an HB pencil for any diagrams or graphs.
Do not use staples, paper clips, glue or correction fluid.
DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **27** printed pages and **1** blank page.

JC18 06_9231_12/RP

[Turn over

1 The curve C is defined parametrically by

$$x = e^t - t, \quad y = 4e^{\frac{1}{2}t}.$$

Find the length of the arc of C from the point where $t = 0$ to the point where $t = 3$. [5]

2 It is given that $f(n) = 2^{3n} + 8^{n-1}$. By simplifying $f(k) + f(k+1)$, or otherwise, prove by mathematical induction that $f(n)$ is divisible by 9 for every positive integer n. [6]

3 The curve C has polar equation $r = \cos 2\theta$, for $-\frac{1}{4}\pi \leqslant \theta \leqslant \frac{1}{4}\pi$.

(i) Sketch C. [2]

(ii) Find the area of the region enclosed by C, showing full working. [3]

....

....

....

....

....

....

....

....

(iii) Find a cartesian equation of C. [3]

4 It is given that the equation

$$x^3 - 21x^2 + kx - 216 = 0,$$

where k is a constant, has real roots a, ar and ar^{-1}.

(i) Find the numerical values of the roots. [6]

(ii) Deduce the value of k. [2]

5 Let $S_n = \sum_{r=1}^{n} (-1)^{r-1} r^2$.

(i) Use the standard result for $\sum_{r=1}^{n} r^2$ given in the List of Formulae (MF10) to show that

$$S_{2n} = -n(2n+1).$$ [4]

(ii) State the value of $\lim_{n \to \infty} \frac{S_{2n}}{n^2}$ and find $\lim_{n \to \infty} \frac{S_{2n+1}}{n^2}$. [4]

6 The curve C has equation

$$y = \frac{x^2 + b}{x + b},$$

where b is a positive constant.

(i) Find the equations of the asymptotes of C. [3]

(ii) Show that C does not intersect the x-axis. [1]

(iii) Justifying your answer, find the number of stationary points on C. [2]

....

....

....

....

....

....

....

....

....

....

....

....

(iv) Sketch C. Your sketch should indicate the coordinates of any points of intersection with the y-axis. You do not need to find the coordinates of any stationary points. [3]

7 Find the particular solution of the differential equation

$$49\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + y = 49x + 735,$$

given that when $x = 0$, $y = 0$ and $\frac{dy}{dx} = 0$. [10]

8 The linear transformation $T : \mathbb{R}^4 \to \mathbb{R}^3$ is represented by the matrix $\mathbf{M}$, where

$$\mathbf{M} = \begin{pmatrix} 1 & 2 & \alpha & -1 \\ 2 & 6 & -3 & -3 \\ 3 & 10 & -6 & -5 \end{pmatrix}$$

and α is a constant. When $\alpha \neq 0$ the null space of T is denoted by K_1.

(i) Find a basis for K_1. [5]

When $\alpha = 0$ the null space of T is denoted by K_2.

(ii) Find a basis for K_2. [3]

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

....

(iii) Determine, justifying your answer, whether K_1 is a subspace of K_2. [2]

....

....

....

....

....

....

....

9 **(i)** Using the substitution $u = \tan x$, or otherwise, find $\int \sec^2 x \tan^2 x \, dx$. [2]

....

....

....

....

....

....

....

....

....

....

....

....

It is given that, for $n \geqslant 0$,

$$I_n = \int_0^{\frac{1}{4}\pi} \sec^n x \tan^2 x \, dx.$$

(ii) Using the result that $\frac{d}{dx}(\sec x) = \tan x \sec x$, show that, for $n \geqslant 2$,

$$(n+1)I_n = (\sqrt{2})^{n-2} + (n-2)I_{n-2}.$$ [5]

....

....

....

....

....

....

....

(iii) Hence find the mean value of $\sec^4 x \tan^2 x$ with respect to x over the interval $0 \leqslant x \leqslant \frac{1}{4}\pi$, giving your answer in exact form. [3]

10 The line l_1 is parallel to the vector $a\mathbf{i} - \mathbf{j} + \mathbf{k}$, where a is a constant, and passes through the point whose position vector is $9\mathbf{j} + 2\mathbf{k}$. The line l_2 is parallel to the vector $-a\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$ and passes through the point whose position vector is $-6\mathbf{i} - 5\mathbf{j} + 10\mathbf{k}$.

(i) It is given that l_1 and l_2 intersect.

(a) Show that $a = -\frac{6}{13}$. [3]

(b) Find a cartesian equation of the plane containing l_1 and l_2. [4]

(ii) Given instead that the perpendicular distance between l_1 and l_2 is $3\sqrt{(30)}$, find the value of a. [5]

11 Answer only **one** of the following two alternatives.

EITHER

(i) Show that if $z = e^{i\theta}$ and $z \neq -1$ then

$$\frac{z-1}{z+1} = i\tan\tfrac{1}{2}\theta. \qquad [3]$$

(ii) Hence, or otherwise, show that if z is a cube root of unity then

$$\frac{z^3 - 1}{z^3 + 1} + \frac{z^2 - 1}{z^2 + 1} + \frac{z - 1}{z + 1} = 0.$$ [5]

(iii) Hence write down three roots of the equation

$$(z^3 - 1)(z^2 + 1)(z + 1) + (z^2 - 1)(z^3 + 1)(z + 1) + (z - 1)(z^3 + 1)(z^2 + 1) = 0$$

and find the other three roots. Give your answers in an exact form. [6]

OR

It is given that **e** is an eigenvector of the matrix **A**, with corresponding eigenvalue λ.

(i) Write down another eigenvector of **A** corresponding to λ. [1]

....

....

....

(ii) Write down an eigenvector and corresponding eigenvalue of $\mathbf{A}^n$, where n is a positive integer. [2]

....

....

....

Let $\mathbf{A} = \begin{pmatrix} 3 & 0 & 0 \\ 2 & 7 & 0 \\ 4 & 8 & 1 \end{pmatrix}$.

(iii) Find a matrix **P** and a diagonal matrix **D** such that $\mathbf{A}^n = \mathbf{PDP}^{-1}$. [7]

....

....

....

....

....

....

....

....

....

....

....

....

(iv) Determine the set of values of the real constant k such that

$$\sum_{n=1}^{\infty} k^n(\mathbf{A}^n - k\mathbf{A}^{n+1}) = k\mathbf{A}.$$ [4]

Additional Page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

BLANK PAGE